Graduate Summer Camp Mathematics Examination

Topics and Questions

The topics of the examination consist of the following:

- Single-variable Calculus
- Linear Algebra
- Probability and Statistics

Detailed descriptions of the topics will be given below. For the examination, you must answer all questions in each topic.

Detailed Topic Descriptions

1) Single-variable Calculus
   - Mathematical induction
   - Rational numbers, real numbers, complex numbers (addition, multiplication, modulus, conjugate, polar form, Euler's relation)
   - Real functions (examples and graphs)
   - Limits and continuity (epsilon-delta definition, techniques for finding limits, continuity via limits of sequences, properties of continuous functions on closed intervals)
   - Differentiation (product rule, chain rule, quotient rule, mean value theorem, l'Hopital's rule, finding extrema, convexity, inverse functions)
   - Sketching graphs of functions (basic principles of curve sketching, asymptotes, extrema, monotonicity, convexity, inflection points)
   - Integration (Riemann/Darboux integral, integration by parts, substitution rule, mean value theorem, fundamental theorem of calculus, improper integrals)

   Suggested Literature:
   - James Stewart, *Calculus* (8th edition), Cengage learning, 2015. ISBN: 978-1-285-74062-1

2) Linear Algebra
   - Matrix algebra (multiplication, inverse, transpose, block matrices)
   - Determinants (Laplace expansion, interpretation as volume of parallelepipeds, solvability of systems of algebraic equations via determinants, Cramer's rule)
   - Vector spaces (linear independence, span, basis, dimension)
   - Normed and Inner product spaces (norm, inner product, Cauchy-Schwarz inequality, induced norm, orthogonality, orthonormal basis, Gram-Schmidt orthonormalization)
   - Linear maps (matrix representation of linear maps in finite-dimensional spaces, kernel, range, dimension formula, adjoint in inner product spaces, change of basis)
   - Eigenvalue Problem (eigenvalues, eigenvectors, diagonalization, spectral theorem for symmetric matrices, similarity, generalized eigenvectors, Jordan form)

   Suggested Literature:

- Sheldon Axler, *Linear Algebra Done Right*, (3rd Edition), Springer 2015. ISBN 978-3-319-11079-0
- Sheldon Axler, 线性代数应该这样学(第 3 版) [Linear Algebra Done Right, (3rd Edition)], 人民邮电出版社 2016. ISBN 7115431787

3) Probability and Statistics

- Basic concepts in probability (Elementary probability, conditional probability, Bayes's Theorem)

- Discrete and continuous random variables (density functions, expectation, variance, joint distributions, covariance and correlation; geometric, binomial, Pascal, hypergeometric and Poisson distributions; exponential, gamma and normal distributions)

- Descriptive statistics (stem-and-leaf diagrams, histograms, boxplots)

- Estimation (point estimators, method of moments, maximum likelihood method, sample mean and sample variance, T distribution, chi-squared distribution, interval estimation)

- Hypothesis testing (null hypothesis significance testing, P-value, Type I and II error probabilities, operating characteristic curves, acceptance sampling)

- Various hypothesis tests (Z-test, T test, chi-squared test, sign test, Wilcoxon signed rank test, test for proportions, comparison of two variances, comparison of two means with known and unknown variances, pooled and paired tests, Pearson chi-squared test for goodness of fit and contingency tables)

- Regression (simple linear regression, polynomial regression and multiple regressors, confidence intervals and hypothesis tests for regression parameters and estimated means, test for lack of fit, correlation and R^2 value, model selection)

Suggested Literature:

- Douglas Montgomery and George Runger, *Applied Statistics and Probability for Engineers*, 5th Edition, Wiley 2011. ISBN 978-0-470-05304-1