Graduate Summer Camp Mathematics Examination

Topics and Questions

The topics of the examination consist of the following:

- Single-variable Calculus
- Multi-variable Calculus
- Differential Equations
- Linear Algebra
- Probability and Statistics

Detailed descriptions of the topics will be given below. For the examination, each applicant must select three out of the five topics listed above. The choice is independent from the major applied for. The applicant must answer all questions in each topic. If more than three topics are worked on, only the first three selected topics will be considered and all other work will be ignored.

Detailed Topic Descriptions

1) Single-variable Calculus
    - Mathematical induction
    - Rational numbers, real numbers, complex numbers (addition, multiplication, modulus, conjugate, polar form, Euler's relation)
    - Real functions (examples and graphs)
    - Limits and continuity (epsilon-delta definition, techniques for finding limits, continuity via limits of sequences, properties of continuous functions on closed intervals)
    - Differentiation (product rule, chain rule, quotient rule, mean value theorem, l'Hopital's rule, finding extrema, convexity, inverse functions)
    - Sketching graphs of functions (basic principles of curve sketching, asymptotes, extrema, monotonicity, convexity, inflection points)
    - Sequences of real numbers (convergence, boundedness, monotonicity, subsequences)
    - Series of real numbers (geometric series, harmonic series, p-series, comparison test, ratio test, root test, integral test, absolute and conditional convergence, alternating series)
    - Power series (radius of convergence, property of being arbitrarily often differentiable, Taylor series)
    - Integration (Riemann/Darboux integral, integration by parts, substitution rule, mean value theorem, fundamental theorem of calculus, improper integrals)

    Suggested Literature:

    - James Stewart, *Calculus* (8th edition), Cengage learning, 2015. ISBN: 978-1-285-74062-1

2) Multiple-Variable Calculus

- Functions of multiple variables (limits, continuity, differentiability and derivative as linear approximation, partial derivatives and Jacobian, chain rule)
- Curves in R^3 (parametrization, tangent, normal and binormal vectors, curvature, torsion)
- Curvilinear coordinates (polar, cylindrical, spherical)
- Potential functions (gradient, directional derivative, tangent plane, Hessian, extrema, Lagrange multiplier method, line integrals)
- Vector fields (divergence and rotation/curl, line integrals, conservative fields, potential fields)
- Volume integrals in R^2 and R^3 (parametrization of domains, Fubini's theorem and iterated integrals, substitution rule)
- Parametrized surfaces in R^3 (surface area, surface integrals of potential functions and vector fields)
- Theorems of Green, Gauss (divergence theorem) and Stokes

Suggested Literature:

- James Stewart, *Calculus* (8th edition), Cengage learning, 2015. ISBN: 978-1-285-74062-1

3) Differential Equations

- Single, first-order differential equations (explicit equations, separation of variables, homogeneous and inhomogeneous linear equations, transformation of equations to a solvable form, general integral curves as solutions, exact equations and solution by finding a potential function, Euler multiplier / integrating factor)
- Linear algebra (Cramer's rule, eigenvalue problem, diagonalizable matrices, basis of eigenvectors for symmetric matrices, vector space, inner product space)
- Linear systems of first-order equations with constant coefficients (solution methods, fundamental system of solutions, Wronskian, Abel's theorem, solutions to inhomogeneous equations)
- Single, second-order, linear differential equations with constant coefficients (solution formula, damped harmonic oscillations, resonance)
- Laplace transform (solutions of ODEs with constant coefficients, inverse transform via tables, convolution, the Dirac delta function)
- Fourier series
- Series method for solving ODEs (Frobenius method, Bessel functions)

Suggested Literature:

- William E. Boyce, Richard C. DiPrima. *Elementary Differential Equations and Boundary Value Problems* (10th edition), Wiley, ISBN 978-0-470-45831-0

4) Linear Algebra

- Matrix algebra (multiplication, inverse, transpose, block matrices)
- Determinants (Laplace expansion, interpretation as volume of parallelepipeds, solvability of systems of algebraic equations via determinants, Cramer's rule)
- Vector spaces (linear independence, span, basis, dimension)
- Normed and Inner product spaces (norm, inner product, Cauchy-Schwarz inequality, induced norm, orthogonality, orthonormal basis, Gram-Schmidt orthonormalization)
- Linear maps (matrix representation of linear maps in finite-dimensional spaces, kernel, range, dimension formula, adjoint in inner product spaces, change of basis)

- Eigenvalue Problem (eigenvalues, eigenvectors, diagonalization, spectral theorem for symmetric matrices, similarity, generalized eigenvectors, Jordan form)

Suggested Literature:

- Sheldon Axler, *Linear Algebra Done Right*, (3rd Edition), Springer 2015. ISBN 978-3-319-11079-0
- Sheldon Axler, 线性代数应该这样学(第 3 版) [Linear Algebra Done Right, (3rd Edition)], 人民邮电出版社 2016. ISBN 7115431787

5) Probability and Statistics

- Basic concepts in probability (Elementary probability, conditional probability, Bayes's Theorem)
- Discrete and continuous random variables (density functions, expectation, variance, joint distributions, covariance and correlation; geometric, binomial, Pascal, hypergeometric and Poisson distributions; exponential, gamma and normal distributions)
- Descriptive statistics (stem-and-leaf diagrams, histograms, boxplots)
- Estimation (point estimators, method of moments, maximum likelihood method, sample mean and sample variance, T distribution, chi-squared distribution, interval estimation)
- Hypothesis testing (null hypothesis significance testing, P-value, Type I and II error probabilities, operating characteristic curves, acceptance sampling)
- Various hypothesis tests (Z-test, T test, chi-squared test, sign test, Wilcoxon signed rank test, test for proportions, comparison of two variances, comparison of two means with known and unknown variances, pooled and paired tests, Pearson chi-squared test for goodness of fit and contingency tables)
- Regression (simple linear regression, polynomial regression and multiple regressors, confidence intervals and hypothesis tests for regression parameters and estimated means, test for lack of fit, correlation and R^2 value, model selection)

Suggested Literature:

- Douglas Montgomery and George Runger, *Applied Statistics and Probability for Engineers*, 5th Edition, Wiley 2011. ISBN 978-0-470-05304-1