



## Course Syllabus

[VE551]

[Matrix Methods for Signal Processing, Data Analysis and Machine Learning]

[Fall 2021]

### Course Description:

Theory and application of matrix methods to signal processing, data analysis and machine learning. Theoretical topics include subspaces, eigenvalue and singular value decomposition, projection theorem, constrained, regularized and unconstrained least squares techniques and iterative algorithms. Applications such as image deblurring, ranking of webpages, image segmentation and compression, social networks, circuit analysis, recommender systems and handwritten digit recognition.

### Instructor:

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Office hour:

1. Tue. & Thu. 13:50-14:20, CRQ 205
2. Wed. 13:00-14:00, Rm. 402 Long Bin Building

### COURSE OBJECTIVES:

1. To introduce matrix-based signal processing, data analysis and machine learning methods and applications that are useful.
2. To provide mathematical foundations for subsequent signal processing, data analysis and machine learning courses.

### COURSE OUTCOMES:

After completing VE551, students should be able to:

1. Demonstrate understanding of basis properties of vectors and matrices.
1. Carry out matrix factorizations, including eigendecomposition and singular value decomposition (SVD).
2. Demonstrate understanding of the concepts of subspace, basis, dimension, null space and rank.



3. Solve linear least-squares (LLS) estimation problems.
4. Demonstrate understanding of vector and matrix norms.
5. Compute low-rank approximation of a matrix.
6. Apply iterative optimization algorithm to applications.
7. Apply matrix completion to applications.

**Textbook (Author, Book Title, Publisher, Publication Year, ISBN):**

None required. Detailed lecture notes written by Prof. Jeffrey A. Fessler are provided.

**Course Prerequisites:**

VE351(Digital Signal Processing) or graduate standing.

**Course Website:**

On canvas.

**Grading Policy (Assignments %, Project, Exams, etc.):**

1. Homework 20%
2. Midterm exam 1 25%
3. Midterm exam 2 25%
4. Final exam 30%

Requests for re-grades of exams must be submitted in writing within one week of exam return. All questions may be re-graded. Letter grades will be assigned using a curve. The median grade is B+.

**Honor Code Policy:**

- 1) Homework: Homework will be posted on Canvas only. Watermarked solution pdf files will be emailed to each student. Some subset of the problems from each assignment will be graded (possibly all). Solutions will be provided for all problems. You are not allowed to post any of the solutions and codes (including your own solutions and codes) on any website. **ABSOLUTELY NO LATE HOMEWORK ASSIGNMENTS WILL BE ACCEPTED.** The lowest homework score will be automatically dropped. About half of the homework assignment points will be JULIA problems. The goal is to develop both analytical skills and computational skills related to matrix methods.
- 2) Collaboration: You must attempt to solve all homework problems, and implement all computer programs



by yourself. Copying homework solutions from another student or from solutions from previous semesters will be considered violations of the **JI honor code** (<https://www.ji.sjtu.edu.cn/wp-content/uploads/2020/10/JI-Honor-Code-202009.pdf>). However, after making a genuine attempt to solve the homework problems, you are encouraged to discuss the answers with other students currently enrolled in VE551 to check the answers and compare solution approaches. After such a discussion, you may rewrite your answer as long as you do so individually, without referring to the solutions of other students or to solutions from previous terms. Basically, the answers you turn in should reflect your own level of understanding, not someone else's. All solutions submitted must be generated by the person whose name appears on the assignment. If you have questions about this policy, please contact Prof. Long. While collaboration can sometimes be helpful to learning, if overused, it can inhibit the development of your problem solving skills.

- 3) Ethics: Sharing any materials from this class with other individuals not in the class without written instructor permission will be treated as an Honor Code violation. Posting your own solutions (including code) on public sites like github.com is also prohibited. Keep your materials private! In particular, uploading any materials from this class to web sites akin to coursehero.com will be reported to the Honor Council.
- 4) Exam: All students must take all exams during the scheduled times. Exceptions must be approved by Prof. Long, in writing stating why you could not attend (severe disease, for example). You must solve all exam problems by yourself. Copying exam solutions from another student or from solutions from previous semesters will be considered violations of the JI honor code. Tentative schedules of exams:
  - a) Midterm Exam1 on Oct. 19, 2021 from 12:10pm-13:50pm. Venue: TBD  
During the exam you may use any paper materials you brought with you but no electronic devices.
  - b) Midterm Exam2 on Nov. 23, 2021 from 12:10pm-13:50pm. Venue: TBD  
During the exam you may use any paper materials you brought with you but no electronic devices.
  - c) Final Exam in the last week. Venue: TBD  
During the exam you may use any paper materials you brought with you but no electronic devices.



### Teaching Schedule:

Week	NO.	lectures and Exams	Comments
1	1	<ul style="list-style-type: none"> <li>Course policies</li> <li>Introduction to Matrices Motivation, Notation</li> <li>1.2 Matrix structures</li> </ul>	
	2	<ul style="list-style-type: none"> <li>1.3 Multiplication</li> <li>1.4 Orthogonality</li> <li>1.5 Matrix determinant</li> </ul>	
2	3	<ul style="list-style-type: none"> <li>1.6 Eigenvalues</li> <li>1.7 trace</li> <li>1.8 Application: Fields Vector Spaces, Linear Transforms</li> </ul>	
	4	<ul style="list-style-type: none"> <li>*** Matrix decompositions</li> <li>2.1 Spectral theorem (for symmetric matrices)</li> <li>2.2 SVD</li> </ul>	
3	5	<ul style="list-style-type: none"> <li>2.3 The matrix 2-norm or spectral norm</li> <li>2.4 relating SVD and eigendecomposition</li> </ul>	
	6	<ul style="list-style-type: none"> <li>2.5 positive (semi)definite matrices</li> <li>*** Subspaces and rank</li> <li>3.1 subspaces</li> </ul>	
4	7	<ul style="list-style-type: none"> <li>3.2 rank</li> <li>3.3 nullspace / nullity</li> <li>3.4 four fundamental subspaces</li> <li>3.5 orthogonal bases</li> <li>3.6 Application</li> </ul>	
	8	<ul style="list-style-type: none"> <li>*** Linear equations and LS</li> <li>4.1 Linear least-squares (LLS) estimation</li> <li>4.2 Moore-Penrose pseudo-inverse</li> </ul>	
5	9	<ul style="list-style-type: none"> <li>4.3 LLS for under-determined case</li> <li>4.4 Truncated SVD solution</li> </ul>	
6	10	<ul style="list-style-type: none"> <li>4.5 Summary of LLS solutions using SVD</li> </ul>	
	11	<ul style="list-style-type: none"> <li>4.6 Frames and tight frames</li> </ul>	
	12	<ul style="list-style-type: none"> <li>4.7 Projection and orthogonal projection</li> </ul>	
7	13	<ul style="list-style-type: none"> <li>5.1 Vector norms</li> </ul>	
	14	<ul style="list-style-type: none"> <li>5.2 Matrix norms</li> <li>5.3 Convergence of sequences of vectors and matrices</li> </ul>	
8	15	<ul style="list-style-type: none"> <li>5.4 Generalized inverse of a matrix /</li> </ul>	



		<ul style="list-style-type: none"><li>• 5.5 Procrustes analysis</li></ul>	
	16	*** Low-rank approximation <ul style="list-style-type: none"><li>• 6.1 Low-rank approximation via Frobenius norm</li></ul>	
9	17	<ul style="list-style-type: none"><li>• 6.2 Sensor localization application (multidimensional scaling)</li><li>• 6.3 Proximal operators</li></ul>	
	18	<ul style="list-style-type: none"><li>• 6.4 Alternative low-rank formulations</li><li>• 6.5 Choosing the rank or regularization parameter</li></ul>	
10	19	<ul style="list-style-type: none"><li>• 6.6 Relate LR to autoencoders, relate to PCA</li><li>• Subspace learning</li></ul>	
	20	*** Special matrices <ul style="list-style-type: none"><li>• 7.1 Companion matrices</li><li>• 7.2 Circulant matrices</li></ul>	
11	21	<ul style="list-style-type: none"><li>• 7.3 Toeplitz matrices</li><li>• 7.4 Power iteration</li></ul>	
	22	<ul style="list-style-type: none"><li>• 7.5 Nonnegative matrices</li><li>• 7.6 Perron-Frobenius theorems</li></ul>	
12	23	<ul style="list-style-type: none"><li>• 8.1 Convergence rate analysis of PGD</li></ul>	
	24	<ul style="list-style-type: none"><li>• 8.2 Preconditioned steepest descent (PSD)</li></ul>	
13	25	<ul style="list-style-type: none"><li>• 8.3 GD for convex functions</li><li>• 8.4 Machine learning for binary classification</li></ul>	
	26	*** Matrix completion <ul style="list-style-type: none"><li>• 9.1 Measurement model</li><li>• 9.2 Noiseless case</li><li>• 9.3 Noisy case</li></ul>	
14	27	Final Exam	