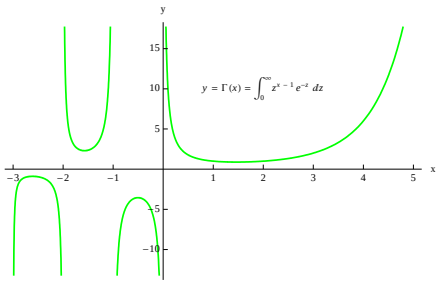
# Course Description

## Vv186 Honors Mathematics II

## Functions of a Single Variable

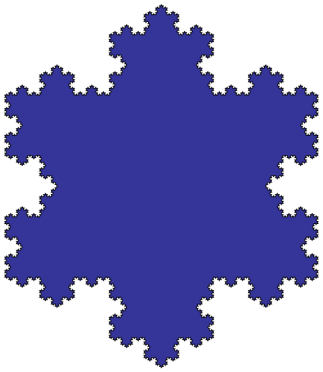
**Prerequisites:** None.

**Course website:** http://umji.sjtu.edu.cn/~horst/teaching/vv186.html



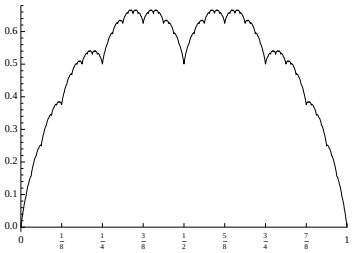

The Euler Gamma function

$\Gamma(n+1) = n!$



The Koch Snowflake

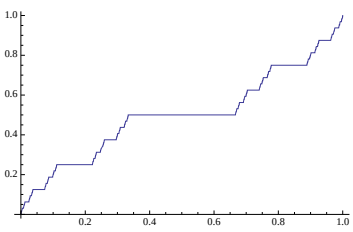
Finite area, infinite circumference




The Van der Waerden function

Everywhere continuous,

nowhere differentiable




The Devil's Staircase

Continuous and almost everywhere

constant, but increasing

**Intended Audience:** ME and ECE undergraduate students.

**Description:** This course consists of four distinct parts. We first fix the rules of language we will be using, giving a brief introduction to mathematical logic and statements in mathematics. We introduce the rational numbers and show various aspects of their insufficiency. We extend them to the set of real numbers and from there to the complex numbers. On the way, we will learn important basic concepts essential for all future mathematical studies.

The second part of the course introduces the concepts of function, convergence and continuity. Starting from the simplest non-finite maps, sequences of real numbers, we familiarize ourselves with the concept of convergence. This is then extended to functions of real variables and the notion of continuity is introduced. Many properties of continuous functions are studied. This leads naturally to the next part of the course.

We next study the linearization of functions and define the derivative of real-valued functions. It subsequently becomes necessary to discuss functions with values that are not real (e.g., complex) and to this end we introduce the notion of normed vector spaces. This allows us to discuss sequences of functions, and we learn about pointwise and uniform convergence. In order to extend the classes of known functions (which heretofore have been limited to rational functions) we discuss series in vector spaces and power series. This leads to the definition of the exponential function and from it the logarithmic and trigonometric functions.

In the final part of the lecture we introduce the notion of the regulated integral. We first define this integral for step functions, where its meaning and value is intuitively clear, then extend it to all functions that can be uniformly approximated by step functions. This procedure allows us to immediately obtain an integral for vector-space-valued functions, which is not possible using the Riemann integral. We will then introduce the Riemann integral as the (equivalent) Darboux integral building on our work with the regulated integral.

After a discussion of the practical issues associated with integration, we continue with a discussion of improper integrals, and applications, such as the definition of the Euler Gamma function and Taylor's theorem with several expressions for the remainder. The culmination will be a proof of Stirling's estimate of the factorial.

**Keywords:** Elements of logic; set theory; properties of rational, real and complex numbers; sequences, convergence, completeness of metric spaces; functions, convergence and continuity; the derivative and applications; normed vector spaces; sequences of functions; series and power series; transcendental functions; the regulated, Darboux and Riemann integrals with applications; Taylor's theorem; Stirling's formula.

**Textbooks:**

- M. Spivak, *Calculus*, 3rd Edition, Cambridge University Press, 2006 (avauilable from the Undergraduate Education Office, Room 315 in the JI Building).

**Syllabus:**

| Lecture | Lecture Subject |
|---|---|
| 1 | Elements of Logic |
| 2 | Set Theory |
| 3 | Natural, Rational and Real Numbers |
| 4 | Complex Numbers |
| 5 | Sequences and Convergence |
| 6 | Sequences and Convergence |
| 7 | Metric Spaces, Real Functions |
| 8 | Limits and Landau Symbols |
| 9 | Properties of Continuous Functions |
| 10 | Properties of Continuous Functions |
| 11 | **First Midterm Exam** |
| 12 | The Derivative |
| 13 | The Derivative |
| 14 | Properties of Differentiable Functions |
| 15 | Properties of Differentiable Functions |
| 16 | Vector Spaces |
| 17 | Sequences of Functions |
| 18 | Series |
| 19 | Series |
| 20 | Power Series |
| 21 | The Exponential and Logarithm Functions |
| 22 | The Trigonometric Functions |
| 23 | **Second Midterm Exam** |
| 24 | Step Functions and Regulated Functions |
| 25 | The Regulated and Darboux Integrals |
| 26 | The Riemann Integral |
| 27 | Integration in Practice |
| 28 | Integration in Practice |
| 29 | Applications of Integration |
| 30 | **Final Exam** |

- The exact location of the exam classes may vary according to the term schedule. The exams in the list above are placed so that the exam topics are precisely those of the preceding lectures.

**Course Grade Components:**

- First midterm exam: 20%
- Second midterm exam: 25%
- Final exam: 30%
- Course work: 25%

**Honor Code Policy:**

**Use of External Sources**
When faced with a particularly difficult hoemwork problem, you may want to refer to other textbooks or online sources such as Wikipedia. Here are a few guidelines:

- Outside sources may treat a similar sounding subject matter at a much more advanced or a much simpler level than this course. This means that explanations you find are much more complicated or far too simple to help you. For example, when looking up the "induction axiom" you may find many high-school level explanations that are not sufficient for our problems; on the other hand, wikipedia contains a lot of information relating to formal logic that is far beyond what we are discussing here.
- If you do use any outside sources to help you solve a homework problem, *you are not allowed to just copy the solution*; this is considered a violation of the Honor Code.
- The correct way of using outside sources is to understand the contents of your source and then to write in your own words and without referring back to the source the solution of the problem. Your solution should differ in style significantly from the published solution. *If you are not sure whether you are incorporating too much material from your source in your solutions, then you must cite the source that you used.*

**Collaboration with other students**
The rules forcollaboration on course work (weekly assignment) problems in this course are quite simple: you must never show any other student your written work. You are not allowed to write down formulas for another student, or to let them see hour homework, or to demonstrate something to them on a blackboard or use any other type of written communication.

You are allowed to talk about the course work, but may not communicate in writing. For example, it is OK to tell another student "I solved this equation by applying l'Hopital's rule." It is not OK to actually show another student the written calculations of how you did this.

The following actions are examples of violations of the Honor Code:

- Showing another student your written solution to a problem.
- Sending a screenshot of your solution via QQ, email or other means to another student.
- Showing another student the written solution of a third student; distributing some student's solution to other students.
- Viewing another student's written solution.
- Copying your solution in electronic form (LaTeX source, PDF, JPG image etc.) to the computer hardware (flash drive, hard disk etc.) of another student. Having another student's solution in electronic form on your computer hardware.

Of course, during exams, no communication of any kind (verbal or written) is allowed!

If you have any questions regarding the application of the Honor Code, please contact me or any of the TAs.