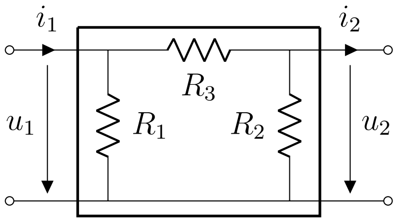
Course Description

Vv285 Honors Mathematics III
Elements of Linear Algebra and Functions of Several Variables

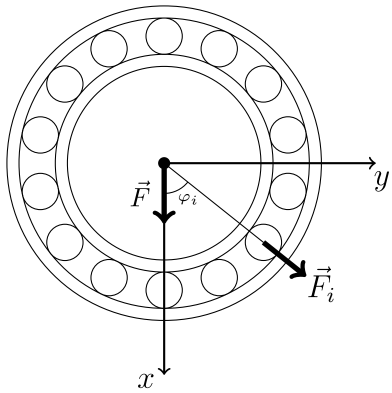
Prerequisites: Vv186 or permission of instructor.

Course website: http://umji.sjtu.edu.cn/~horst/teaching/vv285.html

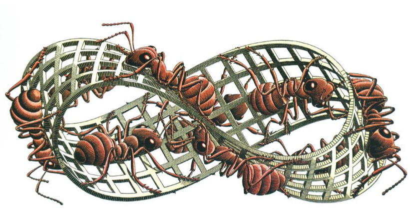
Intended Audience: ME and ECE undergraduate students.



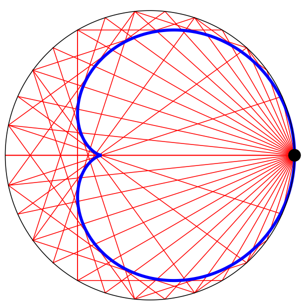
Electric Quadripole in Pi Configuration



Ball Bearing under Load



Möebius Band



Coffee Cup Caustic

Description: The sequence Honors Math Vv186-285-286 is an introduction to calculus at the honors level. It differs from the Applied Calculus sequence in that new concepts are often introduced in an abstract context, so that they can be applied in more general settings later. Most theorems are proven and new ideas are shown to evolve from previously established theory.

The course starts with an introduction to linear algebra, featuring the Gauß-Jordan algorithm for solving systems of equations, the theory of finite dimensional vector spaces, linear maps, matrices and determinants. These concepts are not only essential tools for the following calculus in $\mathbb{R}^n$ but (together with material to be presented in Vv286) comprise the content of an independent course in linear algebra.

The second part of the course starts with a discussion of convergence and continuity before embarking on differential calculus in $\mathbb{R}^n$ and, more generally, in finite-dimensional vector spaces. Derivatives are defined as multilinear maps between vector spaces, and the Jacobian is introduced as their representing matrix. Curves in $\mathbb{R}^n$ and potential functions serve as specific examples and starting points for various applications to engineering and physics. Curve integrals of scalar and vectorial functions are introduced. The second and higher derivatives are defined as multilinear maps, with the Hessian of a potential function a concrete example. Based on the properties of the Hessian, extrema (with and without constraints) are discussed.

The third part focuses on vector fields, surfaces and integration. First, the concepts of divergence and rotation (curl) are introduced together with the corresponding flux and circulation integrals in $\mathbb{R}^2$. A more general discussion of integration of scalar functions on Jordan-measurable sets as well as scalar functions and vector fields on surfaces in $\mathbb{R}^n$ follows. For brevity and simplicity, only parametrized surfaces are treated, i.e., in contradistinction to the theory of curves discussed previously, no definition of surfaces independent of parametrizations is made. The classical theorems of Green in $\mathbb{R}^2$ and Stokes (after a rigorous definition of surfaces with boundary) in $\mathbb{R}^3$ are introduced and proven in simplified cases. The theorem of Gauß (a.k.a. Ostrogradskii or divergence theorem) is proven in $\mathbb{R}^n$. Green's formulas round off the course.

Keywords: Linear systems of equations and the Gauss-Jordan algorithm; finite-dimensional vector spaces, linear independence and bases; scalar products and Gram-Schmidt orthonormalization; linear maps and matrices; determinants; topology of normed spaces; the derivative and applications; curves, potentials and vector fields; higher derivatives and applications; the Riemann integral in n-dimensional space; integration on curves and surfaces; classical theorems of vector analysis in two, three and n dimensions (Green, Stokes and Gauß, respectively).

Syllabus:

Lecture	Lecture Subject
1	Systems of Linear Equations
2	Finite-Dimensional Vector Spaces
3	Inner Product Spaces
4	Linear Maps
5	Matrices
6	Matrices
7	Theory of Systems of Linear Equations
8	Determinants
9	Determinants
10	Determinants
11	**First Midterm Exam**
12	Convergence and Continuity
13	Functions and Derivatives
14	Functions and Derivatives
15	Functions and Derivatives
16	Curves in $\mathbb{R}^n$
17	Curves in $\mathbb{R}^n$
18	Potential Functions
19	The Second Derivative
20	Free Extrema
21	Constrained Extrema
22	**Second Midterm Exam**
23	Vector Fields and Line Integrals
24	Circulation and Flux
25	The Riemann Integral and Measurable Sets
26	Integration in Practice
27	Integration in Practice
28	Surfaces and Surface Integrals
29	Surfaces and Surface Integrals
30	Theorems of Gauß and Stokes
–	**Final Exam**

The exam dates are arranged by subject - in a typical term, the actual exam dates may be shifted somewhat to allow for review time.

Textbooks: What follows is a selection of literature for this course. Find out for yourselves which books are helpful and also do a little research by yourselves for other potentially useful books. You may click directly on the `doi` number to reach a web page for each work. From within the SJTU network, you should be able to freely download an electronic copy.
The first part of this course gives some necessary background in linear algebra. There are several books that might be helpful:

- Jim Hefferon, *Linear Algebra*, Online Book (St Michael's College, Colchester, Vermont, 2001), `http://joshua.smcvt.edu/linearalgebra/`

  A US-style textbook for a first course in linear algebra, with emphasis on calculations.

- Klaus Jänich, *Linear Algebra*, Undergraduate Texts in Mathematics (Springer-Verlag New York, 1994), doi:`10.1007/978-1-4612-4298-7`

  This is the translation of a classic German book that is used as a reference in nearly every course on linear algebra in Germany. Like Spivak's Calculus, it is written in a conversational style, eschewing the definition-theorem-proof-example chain that is so prevalent in serious textbooks.

- Sheldon Axler, *Linear Algebra Done Right*, 3rd ed., Undergraduate Texts in Mathematics (Springer International Publishing, 2015), doi:`10.1007/978-3-319-11080-6`

  This very readable book by a US-American author is the reaction to the prevalent style of linear algebra textbook in the US (see Hefferon above for an example). It is very much in the continental-European tradition of putting structures and relationships before calculations. (You will notice similarities with Jänich's book above.)

- Serge Lang, *Introduction to Linear Algebra*, 2nd ed., Undergraduate Texts in Mathematics (Springer New York, 1986), doi:`10.1007/978-1-4612-1070-2`

  Another serious book on linear algebra, written by an eminent french-born mathematician (now deceased). Although first published in 1986 and in much the same vein as Axler's book, it never became widely popular as an undergraduate textbook in the US, perhaps because it was considered too difficult. But it is actually very readable.

- Serge Lang, *Linear Algebra*, 3rd ed., Undergraduate Texts in Mathematics (Springer New York, 1987), doi:`10.1007/978-1-4757-1949-9`

  A more advanced version of the above book, treating many topics that we do not have time for in this course. Read this for a deeper understanding of linear algebra.

- Rami Shakarchi, *Solutions Manual for Lang's Linear Algebra* (Springer New York, 1996), doi:`10.1007/978-1-4612-0755-9`

  You may find this useful :-)

For the remainder of the course, focussing on multidimensional calculus, there a several books that can be read alongside the lecture notes. However, none of these books covers the course material in exactly the same way as we do and most of them also include much additional material. I am teaching the material in the style of German textbooks, few of which have been translated into English. In the anglo-saxon literature, one is often faced either with (many!) easy calculus books that ignore general concepts or graduate books that are much too advanced.

- Serge Lang, *Calculus of Several Variables*, 3rd ed., Undergraduate Texts in Mathematics (Springer New York, 1987), doi:`10.1007/978-1-4612-1068-9`

  Yes, Serge Lang wrote a lot of undergraduate and graduate textbooks. This one overlaps with some of the course, but takes a different approach to some topics. It is perhaps less abstract than what we do.

- Wendell Fleming, *Calculus of Several Variables*, Undergraduate Texts in Mathematics (Springer New York, 1977), doi:10.1007/978-1-4684-9461-7

  Like Lang's book, this one also overlaps with part of our course. However, the part that does not overlap is more abstract than what we do, so it is useful for further reading.

- J. J. Duistermaat and J. A. C. Kolk, *Multidimensional Real Analysis I* (Cambridge University Press, 2004), doi:10.1017/CBO9780511616716

  J. J. Duistermaat and J. A. C. Kolk, *Multidimensional Real Analysis II* (Cambridge University Press, 2004), doi:10.1017/CBO9780511616723

  This is for the ambitious student. Like Lang, Duistermaat was a very well-known and respected mathematician, and he does everything rigorously and leaves nothing unproven.

Course Grade Components:

- First midterm exam: 30%
- Second midterm exam: 30%
- Final exam: 30%
- Term Project: 10%

The median course grade will be a B+ (or higher, in exceptional cases).

The course work (weekly assignments) will not contribute to the course grade, but each students needs to obtain at least 60% of the total marks of the assignments in order to receive a passing grade for the course. The course work will be completed by groups of 3 students which will remain unchanged throughout the term.

Honor Code Policy:

Students should familiarize themselves with JI's Honor Code, found at

http://umji.sjtu.edu.cn/academics/academic-integrity/honor-code/.

The standard rules for examinations apply.

Furthermore, in group work (both projects and the course work) Section 5 of the Honor Code is fully enforced: any violation of the Honor Code by a group will cause all group members to be sanctioned equally.

Finally, while communication between members of a group is completely unrestricted, communication between groups (even oral communication) is strictly prohibited.

The Teaching Assistants will be happy to answer any any questions regarding the application of the Honor Code.