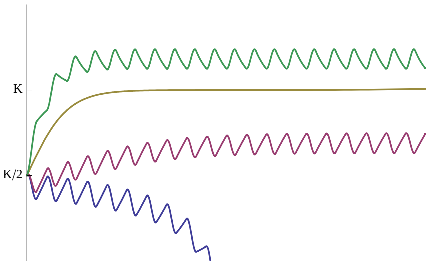
Course Description

Vv286 Honors Mathematics IV

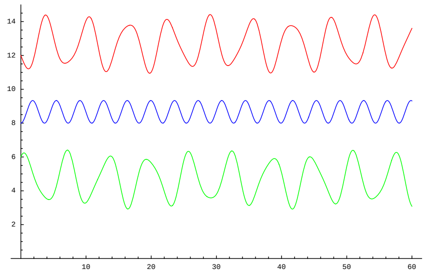
Ordinary Differential Equations and Linear Algebra

Prerequisites: Vv285 or permission of instructor.

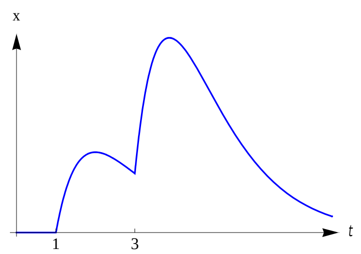
Course website: http://umji.sjtu.edu.cn/personal/horst/teaching/vv286.html



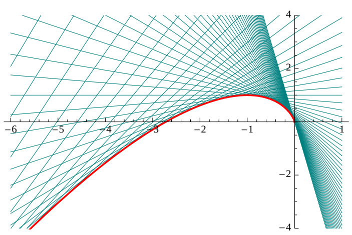
Population dynamics modelled by a Ricatti equation



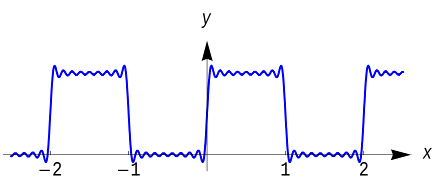
Displacement of a system of three coupled harmonic oscillators



Solution of an impulsive equation



Solutions of the Clairaut equation



Fourier Series of a periodic pulse

Intended Audience: ME and ECE undergraduate students.

Description: This course consists of four distinct parts. In the first part, we will discuss some basic integrable single first order ordinary differential equations. In particular, we will look at several types of explicit and implicit equations, including homogeneous, separable, linear, Bernoulli, Ricatti, Clairaut and d'Alembert equations. We will also look at some concrete modeling examples, such as C-14 dating (using the differential equation for unrestricted growth or decay to zero) and population models (using various flavors of the logistic equation).

In the second part, we will discuss systems of first order equations. After proving a general existence and uniqueness theorem (which also has practical applications) we will introduce some background in linear algebra, in particular eigenvalue problems and matrix similarity. Using these techniques, we will be able to solve constant-coefficient linear systems exactly. Next, we will give a brief introduction to general systems of equations, which touches upon the theory of dynamical systems.

The third part is devoted to integral transform techniques for solving second-order differential equations. We first learn about residual calculus in elementary complex analysis. Since this is also useful elsewhere, and the general concepts of complex analysis will pop up again in more advanced courses, we will devote several lectures to an introduction to complex analysis. Following this, we are able to introduce the Heaviside operator calculus for solving differential equations and from that deduce the Laplace transform technique. We round of this part with a discussion of the analytical Fourier transform and identify the laplace transform as a special case of trhe complex Fourier transform.

In the last part of the course we discuss series-based solutions. the power-series-based Frobenius method leads us to the Bessel functions, which turn out to have a wide range of applications in physics and engineering. We discuss the problem of a hanging chain, self-buckling of a column, diffraction by a circular aperture and more. Series solutions based on trigonometric functions lead to Fourier series, which we view in the general context of orthogonal functions. We also apply this theory to orthogonal Bessel functions and Legendre polynomials and use these to treat some classical partial differential equations by separation of variables.

Keywords: Ordinary differential equations (ODEs) of first order; Systems of first-order equations; the existence and uniqueness theorem of Picard-Lindeloef; eigenvalue problems, diagonalization and the spectral theorem; Jordan normal form; application to linear systems of first-order equations; linear second-order equations; elements of complex analysis and residue theory; the Laplace transform and its inverse with applications to ODEs; power series solutions of ODEs by the Frobenius method; Bessel functions and their applications; Generalized Fourier series; introduction to the classical partial differential equations of physics and some basic solutions by separation of variables.

Syllabus:

Lecture	Lecture Subject
1	Introduction and Explicit First-Order ODEs
2	Separable Equations
3	Linear and Transformable Equations
4	Integral Curves and Implicit Equations
5	Systems of First-Order ODEs
6	The Eigenvalue Problem
7	The Spectral Theorem for Self-Adjoint Matrices
8	The Jordan Normal Form
9	Linear Systems of First-Order ODEs
10	Vibrations
11	**First Midterm Exam**
12	Complex Analysis
13	Properties of Holomorphic Functions
14	Singularities and Poles
15	Residue Calculus
16	The Heaviside Operator Method
17	The Laplace Transform
18	The Laplace Transform
19	The Fourier Transform
20	**Second Midterm Exam**
21	Power Series Solutions to Second Order ODEs
22	Power Series Solutions to Second Order ODEs
23	Applications of Bessel Functions
24	Applications of Bessel Functions
25	Orthonormal Functions
26	Fourier Series
27	Boundary Value Problems
28	The Wave and Heat Equations
29	The Wave and Heat Equations
30	**Final Exam**

Textbooks:

[B] M. Braun, *Differential Equations and their Applications*

[W] W. Walter, *Ordinary Differential Equations*

[J] K. Jänich, *Linear Algebra*

[S] E. M. Stein and R. Shakarchi, *Complex Analysis*

[E] G. Evans, J. Blackledge and P. Yardley, *Analytic Methods for Partial Differential Equations*

[K] B. Korenev, *Bessel Functions and their Applications*

Course Grade Components:

- First midterm exam: 25%
- Second midterm exam: 25%
- Final exam: 25%
- Course work: 25%

Honor Code Policy:

Use of External Sources
When faced with a particularly difficult hoemwork problem, you may want to refer to other textbooks or online sources such as Wikipedia. Here are a few guidelines:

- Outside sources may treat a similar sounding subject matter at a much more advanced or a much simpler level than this course. This means that explanations you find are much more complicated or far too simple to help you. For example, when looking up the "induction axiom" you may find many high-school level explanations that are not sufficient for our problems; on the other hand, wikipedia contains a lot of information relating to formal logic that is far beyond what we are discussing here.
- If you do use any outside sources to help you solve a homework problem, *you are not allowed to just copy the solution*; this is considered a violation of the Honor Code.
- The correct way of using outside sources is to understand the contents of your source and then to write in your own words and without referring back to the source the solution of the problem. Your solution should differ in style significantly from the published solution. *If you are not sure whether you are incorporating too much material from your source in your solutions, then you must cite the source that you used.*

Collaboration with other students
The rules forcollaboration on course work (weekly assignment) problems in this course are quite simple: you must never show any other student your written work. You are not allowed to write down formulas for another student, or to let them see hour homework, or to demonstrate something to them on a blackboard or use any other type of written communication.

You are allowed to talk about the course work, but may not communicate in writing. For example, it is OK to tell another student "I solved this equation by applying l'Hopital's rule." It is not OK to actually show another student the written calculations of how you did this.

The following actions are examples of violations of the Honor Code:

- Showing another student your written solution to a problem.
- Sending a screenshot of your solution via QQ, email or other means to another student.
- Showing another student the written solution of a third student; distributing some student's solution to other students.
- Viewing another student's written solution.
- Copying your solution in electronic form (LaTeX source, PDF, JPG image etc.) to the computer hardware (flash drive, hard disk etc.) of another student. Having another student's solution in electronic form on your computer hardware.

Of course, during exams, no communication of any kind (verbal or written) is allowed!

If you have any questions regarding the application of the Honor Code, please contact me or any of the TAs.